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Effect of Modal Filter Errors on Vibration Control Characteristics

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When designing a control system for vibration suppression of flexible structures using modal control strategy, one must know the modal displacements and velocities of the controlled modes. If the vibration control forces are designed based on inaccurate modal states, the closed-loop performance of the vibration control system will be degraded depending on the extent of the modal filter errors. In this study, the effect of modal filter errors on the vibration control characteristics of flexible structures is analyzed for IMSC (Independent Modal Space Control). A Lyapunov asymptotic stability condition that depends on the magnitude of the modal filter errors is derived. The extent of the response deviation of the closed-loop system is also derived and evaluated using operator techniques. The extent of the response deviation is found to be proportional to the magnitude of the modal filter errors.

Key Words: Modal Filter Errors, Vibration Control, Asymptotic Stability Condition, Observation Spillover.

1. Introduction

In order to suppress vibration of flexible structures such as space structures using modal control method, the modal displacements and velocities must be known. In other words, the calculation of the feedback control forces for vibration suppression requires accurate knowledge of modal states. Since the vibration sensors measure the actual displacements, additional work is needed to convert the sensor readings into the knowledge of modal states. This can be done by using either observers or modal filters. If observers are used to estimate the modal states for the controlled modes of the discretized model of the system, observation spillover due to sensors and control spillover due to a finite number of actuators may cause the closed-loop system to become unstable. If the modal filters based on the expansion theorem is used, however, the observation spillover can be minimized by converting the sensor readings into a distributed output. The output accuracy can be further enhanced by increasing the number of sensors employed. The modal filter method was originally proposed by Meirovitch and Baruh (Meirovitch and Baruh, 1981, 1982), with later refinements (Meirovitch and Baruh, 1985, Choe and Baruh, 1983). However, errors can arise when using the modal filters. The sources of error are system parameter uncertainty, inexact system eigenfunctions, the use of interpolation functions, and a finite number of sensors employed. Hence, the natural question is how the control law designed on the basis of the information furnished by the modal filters containing such errors will influence the performance of the closed-loop system. That is, how robust is the system performance with respect to the modal filter errors ?

Although many methods have been devised to date for vibration control of flexible structures, the main drawback has been that the computational and implementational complexity

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rapidly increases as the number of the controlled modes increases. Meirovitch and coworkers (Meirovitch and Baruh, 1982, Merirovitch, 1980, Meirovitch et al, 1993) have proposed an independent modal space control (IMSC) method in which modal control forces are individually designed for each mode. The method entails using the modal matrix as a transformation matrix to convert the coupled equations of the motion of the system into a set of decoupled equations in modal coordinates. A modal control law can then be designed for one mode at a time based on the information furnished by the respective modal states, and the control system design and implementation can therefore become very simple, irrespective of the total number of system modes to be controlled. The modal control law thus designed can then be converted into the actual control force by using the reverse coordinate transformation. The main drawback of this method is that the number of required actuators should be equal to the number of the controlled modes.

Although a number of studies have dealt with robustness properties of IMSC method (Hwang 1993, 1994, Hwang et al, 1996), the effect of the modal filter errors on the performance characteristics of IMSC has not been reported to date. The main purpose of the present investigation is to undertake such an analysis. First, the effect of the modal estimation error arising from the modal filter errors on the stability of the closed-loop system is investigated; the condition for asymptotic stability in the sense of Lyapunov is derived. Next, an upper bound on the deviation of the vibration control law from the nominal design value due to the modal filter errors is derived using operator techniques. The bound on the response deviation of the closed-loop system is found to be directly proportional to the magnitude of the modal filter errors. The authors believe that the present study is the first reported to date that directly deals with the effect of the modal filter errors on the performance characteristics of the IMSC method.

2. System Design with Modal Filters

Modal filters for estimating modal states of controlled modes can be expressed in the form of

$$\hat{q}_c(t) = Dy(t) \tag{1}$$

$$\hat{\dot{q}}_c(t) = D\dot{y}(t) \tag{2}$$

where $\hat{q}_c(t)$ and $\hat{\dot{q}}_c(t)$ are the estimated modal displacement and velocity vectors of order n, respectively, and $y(t)$ and $\dot{y}(t)$ are the displacement and velocity sensor output vectors of order K, respectively. For computational simplicity and yet without sacrificing generality, the modal filter matrix D for one-dimensional continuous systems can be written as (Meirovitch and Baruh, 1982)

$$D_{ir} = I_{ir}^{(1)} + I_{i-1,r}^{(2)}, \quad i = 1, 2, \cdots, K \quad r = 1, 2, \cdots, n \tag{3}$$

for which

$$I_{ir} = [I_{ir}^{(1)} I_{ir}^{(2)}]^T = h\int_0^1 M(hi - h\xi) \phi_r(hi - h\xi) L(\xi) d\xi I_{Kr}^{(1)} = I_{0r}^{(2)} = 0 \quad 0 \le \xi \le 1.$$

In the above equations, ϕ_r denotes the r-th eigenvector, M the mass operator, h the length of an individual finite element, K the number of sensors, n the number of controlled modes, and $L(\xi)$ the interpolation function vector. Although the slope $y'(t)$ and angular velocity $\dot{y}'(t)$ could also be included in the sensor output for better accuracy, it is not considered here. If enough sensors are used, the desired accuracy can still be obtained from the displacement and velocity sensor outputs only.

From the expansion theorem, the following definition for the displacement sensor output vector can be given (Meirovitch, 1980, Inman, 1989)

$$y_j(t) = w(x_j, t) = \sum_{r=1}^{\infty} \phi_r(x_j) q_r(t), \quad j = 1, 2, \cdots, K \tag{4}$$

Recasting Eq. (4) in matrix form,

$$y(t) = Cq(t) \tag{5}$$

where

$$C_{jr}=\phi_r(x_j),\quad j=1,2,\cdots,K,\ r=1,2,\cdots.$$

Classifying the system modes into the controlled modes $q_c(t)$ and the residual modes $q_R(t)$, the displacement sensor output vector can be cast in the form

$$y(t)=C_C q_C(t)+C_R q_R(t) \tag{6}$$

where

$$C_C=[C_{Cjr}],\ C_{Cjr}=\phi_r(x_j),\ j=1,2,\cdots,K$$
$$r=1,2,\cdots,n$$
$$C_R=[C_{Rjr}],\ C_{Rjr}=\phi_r(x_j),\ j=1,2,\cdots,K$$
$$r=n+1,n+2,\cdots$$

and the velocity sensor output vector can be written as

$$\dot{y}(t)=C_C\dot{q}_C(t)+C_R\dot{q}_R(t) \tag{7}$$

Substituting Eq. (6) and (7) into Eq. (1) and (2), respectively, we have

$$\hat{q}_C(t)=DC_C q_C(t)+DC_R q_R(t) \tag{8}$$
$$\hat{\dot{q}}(t)=DC_C\dot{q}_C(t)+DC_R\dot{q}_R(t) \tag{9}$$

If the matrix D is exact, the modal filter should provide accurate estimates of the modal states, and, $\hat{q}_c(t)$, $\hat{\dot{q}}_c(t)$ will be identical to $q_c(t)$ and, $\dot{q}_c(t)$, respectively. The required condition criteria can be expressed as

$$DC_C=I_C,\ DC_R=0$$

where I_C is the unit matrix of order n. If D is not exact, however, the modal filter output will contain errors, and $\hat{q}_C(t)$ and $\hat{\dot{q}}_C(t)$ will deviate from $q_C(t)$ and $\dot{q}_C(t)$, respectively. If the estimates containing errors are used in feedback, the closed-loop system performance will be degraded.

Let us now consider the following modal equations for the controlled modes

$$\ddot{q}_r(t)+\omega_r^2 q_r(t)=f_r(t),\ r=1,2,\cdots,n \tag{10}$$

where $q_r(t)$ and $f_r(t)$ denote r-th modal displacement and modal force, respectively. The modal force is given by

$$f_r(t)=\int_D \phi_r f(x,t)\,dD,\ r=1,2,\cdots,n \tag{11}$$

where $f(x,t)$ is the actual distributed force. Applying vector notation, Eq. (10) can be expressed as

$$I_C\ddot{q}_C(t)+\Lambda_C q_C(t)=f(t) \tag{12}$$

for which

$$\Lambda_C=diag[\omega_1^2\omega_2^2\cdots\omega_n^2],$$
$$f(t)=[f_1(t)f_2(t)\cdots f_n(t)]^T$$

Let us now apply the IMSC method to design a control law for independent control of each mode. Since the r-th modal force $f_r(t)$ depends only on r-th modal states $q_r(t)$ and $\dot{q}_r(t)$, the following equation can be used

$$f_r(t)=-k_{Pr}q_r(t)-k_{Vr}\dot{q}_r(t),\ r=1,2,\cdots,n \tag{13}$$

Applying optimal control theory furnishes us with the following expressions for the feedback gains k_{Pr} and k_{Vr}:

$$k_{Pr}=\omega_r(-\omega_r+\sqrt{\omega_r^2+R_r^{-1}}),\quad r=1,2,\cdots,n$$
$$k_{Vr}=\{2\omega_r(-\omega_r+\sqrt{\omega_r^2+R_r^{-1}})+R_r^{-1}\}^{1/2},$$
$$r=1,2,\cdots,n,$$

where R_r denotes a weighting factor for the control force. Since $\omega_r>0$, the conditions $k_{Pr}>0$ and $k_{Vr}>0$ are also satisfied. Rewriting Eq. (13) in vector form,

$$f(t)=-K_P q_C(t)-K_V\dot{q}_C(t) \tag{14}$$

for which control gain matrices K_P and K_V are positive definite and given by

$$K_P=diag[k_{P1}k_{P2}\cdots k_{Pn}]$$
$$K_V=diag[k_{V1}k_{V2}\cdots k_{Vn}].$$

However, as the modal displacements $q_c(t)$ and modal velocities $\dot{q}_c(t)$ cannot be directly measured, we obtain the estimated modal displacements $\hat{q}_c(t)$ and modal velocities $\hat{\dot{q}}(t)$ by using the modal filters. Thus, the control force of Eq. (14) is replaced by

$$f(t)=-K_P\hat{q}_C(t)-K_V\hat{\dot{q}}_C(t) \tag{15}$$

We now consider the modal filter errors in some detail. The errors can be generated from several sources: The first is the errors in eigenfunctions φ_r. The second is dividing the continuous region into a finite number of elements. The third is the use of the interpolation function. The fourth is errors in system parameters. All possible errors are to be included, and the modal filter equation that includes errors can be written as

$$\hat{q}_C(t) = (D + \Delta D)\, y(t) \quad (16)$$
$$\hat{\dot{q}}(t) = (D + \Delta D)\, \dot{y}(t) \quad (17)$$

where D represents the exact modal filter matrix, and ΔD denotes a matrix containing the modal filter errors. Substituting Eq. (6) and (7) into Eq. (16) and (17) respectively, we obtain the following equations

$$\hat{q}_C(t) = Dy(t) + \Delta DC_C q_C(t) + \Delta DC_R q_R(t) \quad (18)$$
$$\hat{\dot{q}}_C(t) = D\dot{y}(t) + \Delta DC_C \dot{q}_C(t) + \Delta DC_R \dot{q}_R(t) \quad (19)$$

Since D denotes the exact modal filter matrix, we have $Dy(t) = q_C(t)$ and $D\dot{y}(t) = \dot{q}_C(t)$. Hence, Eqs. (18) and (19) can be rewritten

$$\hat{q}_C(t) = (I_C + \Delta DC_C)\, q_C(t) + \Delta DC_R q_R(t) \quad (20)$$
$$\hat{\dot{q}}(t) = (I_C + \Delta DC_C)\, \dot{q}_C(t) + \Delta DC_R \dot{q}_R(t) \quad (21)$$

For the case of no modal filter errors, the closed-loop equations for the controlled modes can be written as

$$I_C \ddot{q}_C(t) + K_V \dot{q}_C(t) + [\Lambda_C + K_P]\, q_C(t) = 0 \quad (22)$$

When the modal filter errors are included, however, combining Eqs. (15), (20), and (21) yields the following expression:

$$\begin{aligned} f(t) = &-K_P(I_C + \Delta DC_C)\, q_C(t) \\ &- K_V(I_C + \Delta DC_C)\, \dot{q}_C(t) - K_P \Delta DC_R \dot{q}_R(t) \\ &- K_V \Delta DC_R \dot{q}_R(t) \end{aligned} \quad (23)$$

Substituting Eq. (23) into Eq. (12) and rearranging, the closed-looping equations become

$$\begin{aligned} &I_C \ddot{q}(t) + K_V(I_C + \Delta DC_C)\, \dot{q}_C(t) \\ &+ [\Lambda_C + K_P(I_C + \Delta DC_C)]\, q_C(t) \\ &= -K_V \Delta DC_R \dot{q}_R(t) - K_P \Delta DC_R q_R(t) \end{aligned} \quad (24)$$

When modal filter errors are present, the terms in the matrix and the residual modes on the right-hand side of the above equation can serve as a source of continued excitation. Furthermore, Eq. (24) becomes coupled due to the nondiagonality of the ΔDC_C matrix, and the modes can no longer be considered independent. Also, the introduction of observation spillover, in which the residual modes contaminate the measurements of the controlled modes, can significantly influence the system performance. For a complete analysis of the stability and performance of the closed-loop system, the equations governing the residual modes need to be considered in conjunction with those for the controlled modes, further complicating the analysis. Since the primary focus of the present study is to determine the effect of the modal filter errors on system performance, the observation spillover can be assumed to be eliminated for the present. Since the residual modes are generally composed of high-frequency signals, the observation spillover can usually be eliminated by passing the sensor output through a low-pass filter. Hence, the resulting closed-loop system equations can be simpled to the following form

$$\begin{aligned} &I_C \ddot{q}_C(t) + K_V(I_C + \Delta DC_C)\, \dot{q}_C(t) \\ &+ [\Lambda_C + K_P(I_C + \Delta DC_C)]\, q_C(t) = 0 \end{aligned} \quad (25)$$

The corresponding block diagram is given in Fig. 1.

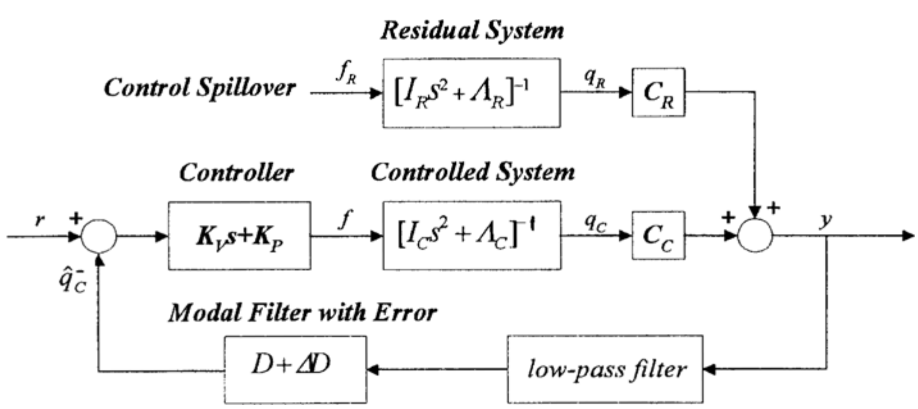

Fig. 1 Block diagram of closed-loop system with modal filter.

3. Stability Analysis of the Closed-loop System

The stability of the closed-loop system is now considered. The asymptotic stability criteria for systems containing no modal filter errors are given by

$$K_V>0,\ \Lambda_C+K_P>0.$$

Since K_P, K_V, Λ_C mentioned in the previous section are each positive definite, the above criteria are satisfied. In other words, in the absence of the modal filter errors, the system is always asymptotically stable. When modal filter errors are present, however, the asymptotic stability criteria cannot readily be obtained. The reason is that the corresponding closed-loop system equations have asymmetric coefficient matrices. To obtain the asymptotic stability criteria, Eq. (25) is first cast in the form

$$I_C\ddot{q}_C(t)+A\dot{q}_C(t)+Bq_C(t)=0 \tag{26}$$

where

$$A=K_V(I_C+\Delta DC_C),$$
$$B=\Lambda_C+K_P(I_C+\Delta DC_C).$$

As mentioned above, the coefficient matrices A and B are asymmetric due to the presence of modal filter errors. We now define the following Lyapunov function for Eq. (26):

$$V(t)=q_C^T[B+B^T]q_C+[q_C^TA^T+\dot{q}_C^T]\cdot[Aq_C+\dot{q}_C]+\dot{q}_C^T\dot{q}_C \tag{27}$$

Since the second and third terms of the above equations are always positive definite, $V(t)$ is assured to be positive definite if the first terms are positive definite. Expressing the condition in equation form, we get

$$B+B^T>0 \tag{28}$$

Differentiating the Lyapunov function with respect to time and substituting Eq. (26) into $\dot{V}(t)$, we obtain the following result:

$$\dot{V}(t)=-q_C^T[A^TB+B^TA]q_C-q_C^T[B^T-B]\dot{q}_C-\dot{q}_C^T[B-B^T]q_C-\dot{q}_C^T[A+A^T]\dot{q}_C.$$

Rewriting in matrix form, we obtain

$$\dot{V}(t)=-z^TQz \tag{29}$$

where

$$Q=\begin{bmatrix}Q_1 & Q_2^T\\ Q_2 & Q_3\end{bmatrix},\ z=\begin{bmatrix}q_C\\ \dot{q}_C\end{bmatrix}$$
$$Q_1=Q_1^T=A^TB+B^TA,$$
$$Q_2=-Q_2^T=B-B^T \tag{30}$$
$$Q_3=Q_3^T=A+A^T$$

If Q in Eq. (29) is positive definite, then $\dot{V}(t)$ is negative definite and the system is asymptotically stable. In order to obtain a simpler formulation of the condition for positive definiteness of Q, we introduce the following transformation:

$$z=Ty \tag{31}$$

where

$$T=\begin{bmatrix}I & -Q_1^{-1}Q_2^T\\ 0 & I\end{bmatrix},\ y=\begin{bmatrix}y_1\\ y_2\end{bmatrix}.$$

Equation (29) can then be cast in the form

$$\dot{V}(t)=-y^TT^TQTy=-y_1^TQ_1y_1-y_2^T[Q_3-Q_2Q_1^{-1}Q_2^T]y_2 \tag{32}$$

A simple inspection of the above equation reveals that if Q_1 and $Q_3-Q_2Q_1^{-1}Q_2^T$ are both positive definite, then $\dot{V}(t)$ is negative definite. The conditions for $V(t)$ to be a Lyapunov function can be summarized as given below:

$$B+B^T>0 \tag{33a}$$
$$Q_1>0 \tag{33b}$$
$$Q_3-Q_2Q_1^{-1}Q_2^T>0 \tag{33c}$$

If the above conditions are satisfied, the system will be asymptotically stable even in the presence of modal filter errors. In summary, it can be seen that the coefficient matrices A and B are asymmetric due to the modal filter errors ΔDC_c, and condition (33) may be violated, for which the system can become unstable.

4. Effect of Modal Filter Errors on Vibration Response Characteristics

The effect of modal filter errors on the system response is considered in this section. Let $e_C(t)$ be the error between the displacement vectors of the controlled mode with the exact modal filters and the modal filters containing errors:

$$e_c(t)=[e_1(t)\,e_2(t)\cdots e_n(t)]^T.$$

Subtracting Eq. (25) from Eq. (22), the governing equation for $e_c(t)$ can be written as

$$I_c\ddot{e}_c(t)+K_V\dot{e}_c(t)+(\Lambda_c+K_P)e_c(t)=K_V\Delta DC_c\dot{q}_c(t)+K_P\Delta DC_cq_c(t) \quad (34)$$

The above equation can be used to predict the response error $e_c(t)$ that results from designing vibration controllers without regard to the modal filter errors. It can be seen that the forcing term for $e_c(t)$ is furnished by ΔD. In the absence of modal filter errors, we have $\Delta D=0$, and consequently, $e_c(t)=0,\ t\geq 0$.

To obtain an upper bound on $e_c(t)$, we define the following L_∞-norm:

$$\|h(t)\|=\max_{1\leq i\leq n}\sup_{t\geq 0}|h_i(t)| \quad (35)$$

where $h(t)$ denotes an arbitrary function of order n, $h(t)=[h_1(t)\,h_2(t)\cdots h_n(t)]^T$. Setting the initial conditions, $e(t)=\dot{e}(t)=0$, we apply the Laplace transform to Eq. (34) to obtain

$$\bar{e}_c(s)=[I_cs^2+K_Vs+\Lambda_c+K_P]^{-1}(K_Vs+K_P)\cdot\Delta DC_c\bar{q}_c(s) \quad (36)$$

where $\bar{e}_c(s)$ and $\bar{q}_c(s)$ denote the Laplace transforms of $e_c(t)$ and $q_c(t)$, respectively. We can recast the above equation in the form

$$\bar{e}_c(s)=\bar{H}(s)\,\bar{q}_c(s) \quad (37)$$

in which each element of the $n\times n$ matrix $\bar{H}(s)$ is given by

$$\bar{H}_{ij}(s)=\frac{k_{Vi}s+k_{Pi}}{s^2+k_{Vi}s+\omega_i^2+k_{Pi}}d_{ij},\quad i,\ j=1,\ 2,\ \cdots,\ n.$$

and d_{ij} denotes ij-th element of ΔDC_c. Applying the inverse Laplace transform to Eq. (36), we obtain the following expression involving the convolution integral

$$e_c(t)=[H^*q_c](t)=\int_0^t H(t-\tau)\,q_c(\tau)\,d\tau \quad (38)$$

where $H(t)=L^{-1}[\bar{H}(s)]$. To obtain a simple expression for Eq. (38) we define the following linear operator

$$\hat{H}(q_c)=[H^*q_c](t) \quad (39)$$

From Eqs. (38) and (39), an upper bound on $e_c(t)$, $\|e_c\|$, can be given by

$$\|e_c\|=\|\hat{H}(q_c)\|\leq\|\hat{H}\|\|q_c\| \quad (40)$$

where $\|\hat{H}\|$ is the L_∞-induced norm of linear operator $\hat{H}$. Invoking an identity given by Desoer and Vidyasaga (Desoer and Vidyasaga, 1975), we obtain for $\|\hat{H}\|$,

$$\|\hat{H}\|=\max_{1\leq i\leq n}\sigma_i\int_0^\infty|h_i(\tau)|d\tau \quad (41)$$

in which σ_i, $i=1,\ 2,\ \cdots,\ n$ denote the i-th row sum of the absolute value of matrix, ΔDC_c, i. e., $\sigma_i=\sum_{j=1}^{n}|d_{ij}|$, and the function $h_i(t)$ is given by

$$h_i(t)=L^{-1}\left(\frac{k_{Vi}s+k_{Pi}}{s^2+k_{Vi}s+\omega_i^2+k_{Pi}}\right)=\frac{k_{Vi}}{\hat{\omega}_i}\sqrt{\frac{\alpha_i^2-2\alpha_i\zeta_i\hat{\omega}_i+\hat{\omega}_i^2}{1-\zeta_i^2}}\exp(-\zeta_i\hat{\omega}_it)\sin(\hat{\omega}_i\sqrt{1-\zeta_i^2}\,t+\theta_i) \quad (42)$$

where

$$\hat{\omega}_i=\sqrt{\omega_i^2+k_{Pi}},$$
$$\zeta_i=\frac{k_{Vi}}{2\sqrt{\omega_i^2+k_{Pi}}},\ (\zeta_i<1),$$
$$\theta_i=\tan^{-1}\frac{\hat{\omega}_i\sqrt{1-\zeta_i^2}}{\alpha_i-\zeta_i\hat{\omega}_i},$$
$$\alpha_i=k_{Pi}/k_{Vi},\ i=1,\ 2,\ \cdots,\ n$$

Substituting Eq. (42) in Eq. (41) and integrating, we obtain

$$\int_0^\infty|h_i(\tau)|d\tau=\frac{k_{Pi}}{\hat{\omega}_i^2}+\frac{k_{Vi}}{\hat{\omega}_i^2}X_{1i}\left[\frac{\exp(X_{2i}+X_{3i})}{1-\exp(X_{3i})}\right] \quad (43)$$

where

$$X_{1i}=2\sqrt{\alpha_i^2-2\alpha_i\zeta_i\hat{\omega}_i+\hat{\omega}_i^2},\ i=1,\ 2,\ \cdots,\ n,$$
$$X_{2i}=\frac{\zeta_i}{\sqrt{1-\zeta_i^2}}\tan^{-1}\frac{\hat{\omega}_i\sqrt{1-\zeta_i^2}}{\alpha_i-\zeta_i\hat{\omega}_i},\ i=1,\ 2,\ \cdots,\ n,$$
$$X_{3i}=-\frac{\zeta_i\pi}{\sqrt{1-\zeta_i^2}},\ i=1,\ 2,\ \cdots,\ n.$$

Rearranging Eq. (41), we obtain

$$\|\hat{H}\|=\max_{1\leq i\leq n}\frac{\sigma_i}{\hat{\omega}_i^2}\left[k_{Pi}+k_{Vi}X_{1i}\frac{\exp(X_{2i}+X_{3i})}{1-\exp(X_{3i})}\right] \quad (44)$$

Incorporating the above results allows us to rewrite Eq. (40) in the form

$$\|e_c\|=\max_{1\leq i\leq n}\sup_{t\geq 0}|e_i(t)|\leq\left[\max_{1\leq i\leq n}\frac{\sigma_i}{\hat{\omega}_i^2}\left(k_{Pi}+k_{Vi}X_{1i}\frac{\exp(X_{2i}+X_{3i})}{1-\exp(X_{3i})}\right)\right]\|q_c\| \quad (45)$$

The above inequality can be cast in the form of $\|e_c\| \leq m\|q_c\|$ where m takes the place of $\|\hat{H}\|$ (Stakhold, 1979). $\|\hat{H}\|$ is dependent on σ_i and the natural frequency ω_i. Thus, if the modal filter errors are small, i. e., $\sigma_i \ll 1$, $i=1, 2, \cdots, n$, $\|\hat{H}\|$ will also be small. Equation (45) also reveals that a linear relationship between $\|e_c\|$ and the magnitude of the modal filter error exists, while the proportionality constant is determined by the type of control method. In summary, the presence of modal filter errors implies an error in the vibration control response, with its magnitude bound by $\|\hat{H}\|\|q_c\|$. Therefore, $e_c(t)$ always lies within a band defined by $\pm\|\hat{H}\|\|q_c\|$.

5. Examples

To demonstrate the applicability of the results of the previous sections, a Bernoulli-Euler beam clamped at one end is considered. The governing equation is given by

$$EI[\partial^4 w(x, t)/\partial x^4] + M[\partial^2 w(x, t)/\partial t^2] = f(x, t).$$

The mass per unit length $M=1$, dynamic stiffness $EI=10$, and length $l=10$. The boundary conditions are

$$B_1(0)=1, \ B_2(0)=d/dx,$$
$$B_1(l)=d^2/dx^2, \ B_2(l)=d^3/dx^3.$$

Solving for the corresponding eigenvalue problem, the eigenvalues can be obtained from the following transcendental equation

$$\cos(\beta_r l)\cosh(\beta_r l) = -1.$$

Solving the above equation.

$$\beta_r l = [1.8751, 4.6941, 7.8548, 10.9955, 14.1372, 17.2788, \cdots].$$

Hence, the eigenvalues and eigenvectors are given by

$$\lambda_r = \omega_r^2 = \beta_r^4 EI/M, \ r=1, 2, \cdots,$$
$$\phi_r = a_r\{[\sin(\beta_r l) - \sinh(\beta_r l)][\sin(\beta_r x) - \sinh(\beta_r x)] + [\cos(\beta_r l) + \cosh(\beta_r l)][\cos(\beta_r x) - \cosh(\beta_r x)]\},$$

where a_r is a constant vector for normalizing the eigenfunctions and given by

$$a_r = [0.1041, 0.005788, 0.0002453, 0.0000106, 0.0000004585, 0.00000001981, \cdots].$$

The number of controlled modes and actuators are each selected to be six, i. e. $n=6$. The sensor positions are given by

$$x_j = \frac{l}{K-1}(j-1), \ j=1, 2, \cdots, K,$$

where x_j denotes the j-th sensor position and K the number of sensors. The interpolation function is given by

$$L_1(\xi) = \xi, \ L_2(\xi) = 1-\xi.$$

By applying the IMSC method, the optimal feedback gain matrices can be computed as follows:

$$K_p = diag[0.1454, 0.6130, 0.8948, 0.9680, 0.9878, 0.9945],$$
$$K_v = diag[1.5135, 1.7961, 1.9467, 1.9839, 1.9939, 1.9972].$$

The weighting factor R_r used in the computation of the feedback gain matrix has been set at 0.5.

If we assume that the exact system parameters and eigenfunctions are known, then the remaining sources of the modal filter errors are the finite number of sensors and the interpolation functions. Regardless of the sources, the modal filter errors are taken up by ΔD. Therefore, we will confine ourselves to those errors arising from the finite number of sensors only. This allows for simpler computation and yet entails no loss of generality. For theses different values of K, the corresponding ΔDC_c can be computed as given below:

For $K=12$: $\Delta DC_c =$

$$\begin{bmatrix} 0.0006 & -0.0081 & 0.0819 & -0.0258 & 0.0359 & -0.0435 \\ 0.0013 & -0.0091 & -0.0062 & 0.0210 & -0.0233 & 0.0375 \\ 0.0011 & 0.0022 & -0.0309 & -0.0057 & 0.0253 & -0.0218 \\ 0.0008 & 0.0038 & 0.0030 & -0.0652 & -0.0055 & 0.0299 \\ 0.0006 & 0.0026 & 0.0078 & 0.0033 & -0.1103 & -0.0053 \\ 0.0005 & 0.0028 & 0.0045 & 0.0121 & 0.0036 & -0.1641 \end{bmatrix}$$

For $K=7$: $\Delta DC_c =$

$$\begin{bmatrix} 0.0020 & -0.0272 & 0.0637 & -0.0872 & 0.1229 & -0.1516 \\ 0.0043 & -0.0297 & -0.0211 & 0.0724 & -0.0812 & 0.1390 \\ 0.0036 & 0.0075 & -0.0972 & -0.0199 & 0.0947 & -0.0803 \\ 0.0025 & 0.0132 & 0.0102 & -0.1929 & -0.0198 & 0.1414 \\ 0.0022 & 0.0090 & 0.0292 & 0.0120 & -0.2897 & -0.0205 \\ 0.0018 & 0.0103 & 0.0166 & 0.0573 & 0.0137 & -0.2565 \end{bmatrix}$$

For $K=2$: $\Delta DC_C=$

$$\begin{bmatrix} 0.1376 & -1.1376 & 1.1376 & -1.1376 & 1.1376 & -1.1376 \\ 0.1815 & -1.1815 & 0.1815 & -0.1815 & 0.1815 & -0.1815 \\ 0.0648 & -0.0648 & -0.9352 & -0.0648 & 0.0648 & -0.0648 \\ 0.0331 & -0.0331 & 0.0331 & -1.0331 & 0.0331 & -0.0331 \\ 0.0200 & -0.0200 & 0.0200 & -0.0200 & -0.9800 & -0.0200 \\ 0.0134 & -0.0134 & 0.0134 & -0.0134 & 0.0134 & -1.0134 \end{bmatrix}$$

The above matrices reveal that as K increases, the modal filter errors diminish. Applying Eq. (33) to determine stability for the above cases, the following results can be obtained:

For $K=12$ $B+B^T>0$, $Q_1>0$, $Q_3-Q_2Q_1^{-1}Q_2^T>0$,
For $K=7$ $B+B^T>0$, $Q_1>0$, $Q_3-Q_2Q_1^{-1}Q_2^T>0$,
For $K=2$ $B+B^T>0$, $Q_1<0$, $Q_3-Q_2Q_1^{-1}Q_2^T<0$.

For cases $K=12$ and $K=7$, Eq. (33) for asymptotic stability is satisfied. However, for $K=2$, the inequalities $Q<0$ and $Q_3-Q_2Q_1^{-1}Q_2^T<0$ imply that condition (33) cannot be satisfied, and the system is unstable. The system eigenvalues for the cases of the exact modal filter and the modal filter containing errors are shown in Table 1.

The tabulated values reveal that since for a sufficient number of sensors the modal filter errors are small, the eigenvalue deviations are small as well. When the number of sensors is insufficient, however, the modal filter errors increase, and the complex eigenvalues have positive real part, i. e., the system becomes unstable. To summarize, increasing the modal filter errors can degrade the stability of the vibration control system, as attested to in Table 1.

Table 1 Closed-loops system eigenvalues for exact modal filter and modal filters containing errors.

Mode No.	Exact	$K=12$	$K=7$	$K=2$
1	$-0.0727\pm1.2331i$	$-0.0727\pm1.2335i$	$-0.0727\pm1.2345i$	$-0.0480\pm0.8400i$
2	$-0.3065\pm1.4791i$	$-0.3037\pm1.4741i$	$-0.2974\pm1.4627i$	$0.0350\pm1.0636i$
3	$-0.4474\pm2.3565i$	$-0.4336\pm2.3463i$	$-0.4038\pm2.3240i$	$-0.0450\pm1.9986i$
4	$-0.4840\pm4.0456i$	$-0.4524\pm4.0332i$	$-0.3905\pm4.0083i$	$0.0182\pm3.8142i$
5	$-0.4939\pm6.4571i$	$-0.4394\pm6.4440i$	$-0.3509\pm6.4214i$	$-0.0103\pm6.3232i$
6	$-0.4972\pm9.5334i$	$-0.4157\pm9.5201i$	$-0.3698\pm9.5119i$	$0.0068\pm9.4398i$

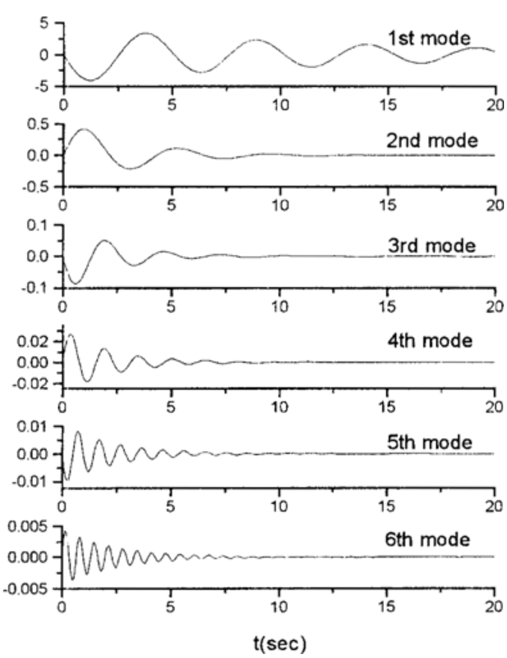

Fig. 2 System Modal Responses for 12 sensors.

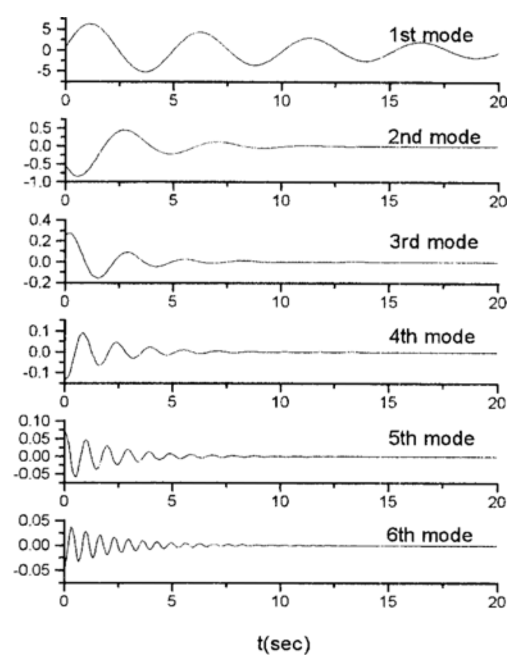

Fig. 3 System Modal Control Forces for 12 sensors.

For the above cases, the time-domain response of the system was been obtained through computer simulation under identical initial conditions. The results for cases $K=12$ and $K=2$ are shown in Figs. 2-7. The modal displacement responses of the system for $K=12$ are shown in Fig. 2, while the corresponding modal control forces are shown in Fig. 3. The resulting time-domain beam displacement is shown in Fig. 4. For $K=2$, the modal displacements are shown in Fig. 5, the modal control forces are shown in Fig. 6, and the time-domain beam displacement is shown in Fig. 7.Figures. 5-7 illustrate that when large modal filter errors are present, the time-domain beam displacement response will diverge. In the presence of modal filter errors, a set of originally decoupled equations obtained by applying the IMSC method becomes a set of coupled equations, and the previously stable modes may become unstable.

We now obtain an upper bound on the vibration control response error. From the previous section

$$\|e_c\| \leq \|\hat{H}\| \|q_c\|.$$

Applying Eq. (44), $\|\hat{H}\|$ can be computed as listed in Table 2. The vibration control response error will always be bound by $\|\hat{H}\|\|q_c\|$, and $e_c(t)$

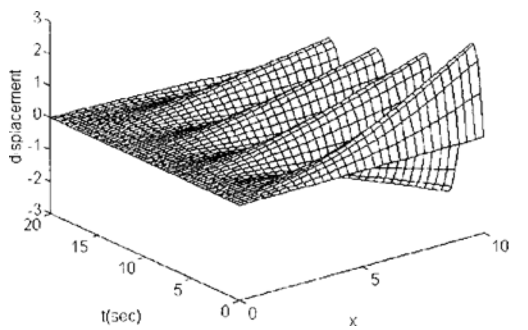

Fig. 4 Beam Displacement Response for 12 sensors.

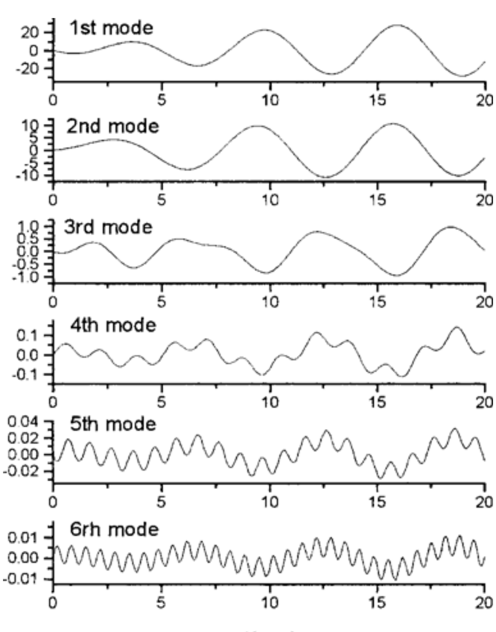

Fig. 5 System Modal Responses for 2 sensors.

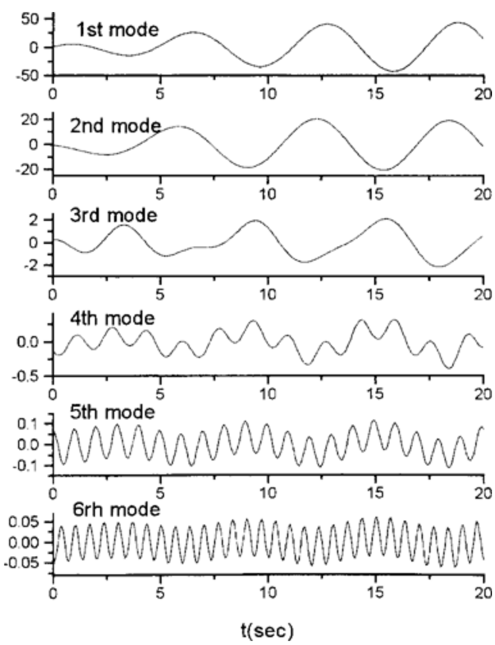

Fig. 6 System Modal Control Forces for 2 sensors.

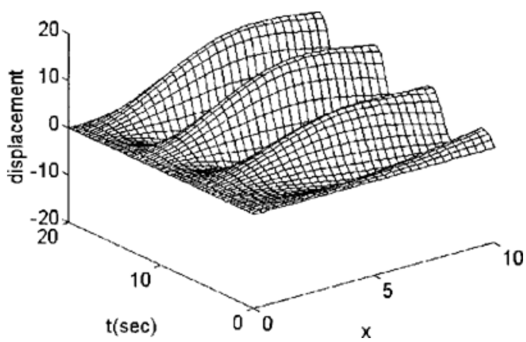

Fig. 7 Beam Displacement Response for 2 sensors.

Table 2 Upper bounds on $\|\hat{H}\|$ for different numbers of sensors.

KK	12	7
$\|\hat{H}\|$	0.1718	0.4322

will always lie within a band defined by $\pm\|\hat{H}\|\|q_c\|$. As the number of sensors decreases, i. e., as the modal filter errors increase, the vibration response error increases in proportion.

6. Conclusions

In the present study, the effect of modal filter errors on the stability and response properties of the vibration control system designed in independent modal space has been analyzed. The modal filter errors arise from the errors in the system parameters, the eigenfunctions and the interpolation functions, as well as from the finite number of sensors employed. Since the computation and subsequent application of the vibration control forces will be based on erroneous information furnished by the modal filters containing the errors, a cririicial issue is whether the vibration control response of the system will be robust with respect to these errors. It is this issue which the present investigation has attempted to address. The principal results can be summarized as follows:

(1) In the presence of modal state estimation errors due to modal filter errors, a Lyapunov asymptotic stability condition (33) for closed-loop vibration control system is derived.

(2) As the magnitude of the modal filter errors increases, the stability characteristics of the closed-loop vibration control system becomes degraded.

(3) For a given modal filter error matrix ΔD, an upper bound (45) on the vibration response is derived using the L_∞-norm.

(4) The upper bound (45) is directly proportional to ΔD, and the proportionality constant is determined by the type of control method applied.

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References

Choe, K. and Baruh, H., 1993, "Sensor Failure Detection in Flexible Structures Using Modal Observers," *Journal of Dynamic Systems, Measurement, and Control,* Vol. 115, Sep., pp. 411~418.

Desoer, C. A. and Vidyasaga, M., 1975, Feedback systems : Input-Output Properties, Academic Press, New York.

Hwang, J. H. and 1993, "Effect of Modal filter Error on the Vibration Control Characteristics," Journal of The Korean Society for Aeronautical and Space Sciences, Vol. 21, No. 2, pp. 37~49.

Hwang, J. H. and Baek, S. H., 1994, "Robustness of Positive Position Feedback Control in the Independent Modal Space," *Journal of Korean Society for Noise and Vibration Engineering,* Vol. 4, No. 2, pp. 177~185.

Hwang, J. H., Kim, D. M. and Lim, K. H., 1996, "Robustness of Natural Controls of Distributed-Parameter Systems," *ASME Journal of Vibration and Acoustics,* Vol. 118, No. 1, pp. 56~63.

Inman, D. J., 1989, Vibration with Control, Measurement, and Stability, Prentice-Hall, Englewood Cliffs, NJ.

Meirovitch, L., 1990, Dynamics and Control of Structures, John Wiley & Sons, New York.

Meirovitch, L., 1980, Computational Methods in Structural Dynamics, Sijthoff & Noordhoff International Publishers B. V., Alphen aan den Rijn, The Netherlands.

Meirovitch, L. and Baruh, H., 1981, "Effect of Damping on Observation Spillover Instability," *Journal of Optimization Theory and Applications,* Vol. 35, No. 1, pp. 31~44.

Meirovitch, L. and Baruh, H., 1982, "On the Problem of Observation Spillover in Distributed-Parameter Systems," *Journal of Optimization Theory and Applications,* Vol. 39, No. 2, pp. 319~341.

Meirovitch, L. and Baruh, H., 1985, "On the Implementation of Modal Filters for Control of

Structures," *Journal of Guidance, Control, and Dynamics,* Vol. 8, No. 6, pp. 707~716.

Meirovitch, L. and Baruh, H., 1982, "Control of Self-Adjoint Distributed-Parameter Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 5, No. 1, pp. 60~66.

Meirovitch, L., Baruh, H. and Oz, H., 1983, "A Comparison of Control Techniques for Large Flexible Systems," J. Guidance and Control, Vol. 6, pp. 302~310.

Stakgold, I., 1979, Green's Functions and Boundary Value Problems, Wiley Interscience, New York.